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Léa Penazzi, Stéphane Blanco, Cyril Caliot, Christophe Coustet, Mouna El-Hafi, Richard A Fournier, Jacques Gautrais, Morgan Sans

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L. Penazzi[*,**,§], S. Blanco[***], C. Caliot[****], C. Coustet[*****], M. El Hafi[*], R. Fournier[***], J. Gautrais[******], and M. Sans[*]

[*]RAPSODEE, UMR CNRS 5302, IMT Mines Albi, Campus Jarlard, Albi, France
[**]PROMES, UPR CNRS 8521, 7 rue du Four Solaire, 66120 Odeillo, France
[***]LAPLACE, UMR CNRS 5213, Université Paul Sabatier, 118 route de Narbonne - 31062 Toulouse, France
[****]LMAP, UMR CNRS 5142, E2S UPPA, Université de Pau et des Pays de l'Adour, 64600 Anglet, France
[*****]Méso-Star, 8 rue des Pêchers, 31410 Longages, France
[******]Centre de Recherches sur la Cognition Animale (CRCA), Centre de Biologie Intégrative (CBI), Université de Toulouse, CNRS, Université Paul Sabatier - Toulouse, France
[§]Correspondence author. Email: lea.penazzi@mines-albi.fr

ABSTRACT The optimization of thermal transfers in engineering systems such as heat exchangers requires the analysis of the influence of heat sources upon the temperature at various positions of interest in the studied system. In order to achieve the resolution of the combined modes of heat transfer through these systems, their couplings and the complex 3D geometries involved need to be integrated with full accuracy. Recent developments in probabilistic formulations in the context of transient combined heat transfer (linearized conduction-radiation-convection) have opened a new route to solving such problems with Monte Carlo (MC) algorithms, using state-of-the-art computer graphics digital libraries to handle complex geometries. To estimate the temperature at a probe point of interest, random paths are generated from its position and propagated through the geometry until a known temperature is reached. From a single MC calculation to sample the path statistics, the Symbolic Monte Carlo (SMC) method is used to express the probe temperature as a linear function of the sources. This function can then be used to estimate the probe temperature for any source values, alleviating the need to repeat Monte-Carlo simulations for each source condition, resulting in greatly reduced computation time. This approach is applied to the case of an open-cavity porous medium and computation time insensitivity to the complexity and fineness of the geometry is demonstrated.

INTRODUCTION

In the context of thermal engineering applications (electronic device cooling, concentrated solar thermal applications, thermal performance of buildings, etc.) geometries are complex (large scale ratios) and the thermal transfers involved, such as conduction, convection or radiation, are coupled. When aiming to achieve parametrical optimization of these applications, the challenge is to develop fast models which, as far as possible, take into account the complexity of the geometry and the combined heat transfer. A common engineering practice when facing these issues is to evaluate the transfer function between the temperature of interest and each source of energy. This function is the *propagator*, which is nothing more than the object that propagates information from the heat sources to the probe temperature of interest. When it comes to evaluating the function in large CAD geometries, standard techniques, such as deterministic methods, tend to be time consuming. Monte Carlo (MC) methods are well known for estimating the propagator with ease in case models requiring large refined geometries, and for solving radiation problems efficiently [Eymet, 2009]. Recent developments in Green's formulation and stochastic processes for combined heat transfer have led to the generation of conducto-convecto-radiative paths to solve such problems with MC algorithms [Fournier, 2016; Caliot, 2017; Ibarrart, 2018]. Random paths are generated from the probe location until a known temperature is reached. Advanced computer graphics tools have recently been developed to solve combined heat transfer in complex geometries and to estimate the temperature at a given location and time for any level of geometrical complexity. With an appropriate storage of the data contained in random paths generated during a single MC calculation, a Symbolic Monte Carlo (SMC) algorithm can be used to estimate the local temperature as a linear function depending on the heat sources and the initial temperature of the thermal problem. This function, as well as the associated uncertainty, can then be used immediately in optimization algorithms, which require a multiple access to the solution for the various values of the different sources. Originally, the

SMC method was introduced as the *Inverse Monte Carlo method* and developed for inverse problems applied to radiative transfer [Dunn, 1981]. It has also recently been used, for example, to identify the radiative properties of heterogeneous materials [Maanane, 2020]. To the best of our knowledge, the SMC method was previously developed only for radiative heat transfer. Recent developments have now extended the use of this method to combined heat transfer [Penazzi, 2019]. Previously, [Villefranque, 2019] showed the insensitivity of computation time to mesh refinements for the resolution of radiative transfer with the MC method in the field of atmospheric radiation. [Ibarrart, 2018] highlighted such insensitivity to geometrical complexity in the case of combined heat transfer resolution with the MC method. In the present paper, such insensitivity to mesh refinement and geometrical complexity is extended to the use of the SMC method. First, a coupled thermal model applied to an open-cavity porous medium is developed, along with the corresponding transfer function obtained with a SMC algorithm. Then, numerical results showing the insensitivity of computation times when evaluating and using the transfer function are presented for different porosities and various mesh refinements.

METHOD

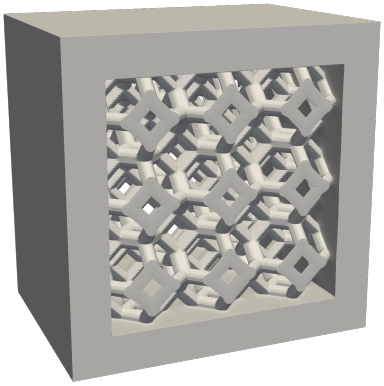

Figure 1: Geometry of the 3D porous medium ($L_x = 150$mm, $L_y = L_z = 200$mm)

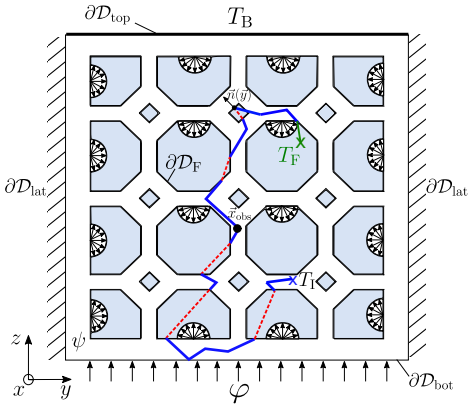

Figure 2: Scheme of the physical problem applied to the porous medium.

The studied configuration (cf. Figure 1) is a heterogeneous 3D porous medium made up of nine kelvin porous cells (3×3 cells) forming an open-porosity medium enclosed in a rectangular-shaped channel which corresponds to a heat exchanger. Conduction occurs in the solid domain $\mathcal{D}$, convection is considered at the solid/fluid interface $\partial\mathcal{D}_{\text{F}}$ and radiation occurs in the fluid domain (transparent). Hence, the corresponding set of equations is :

$$\begin{cases} \rho c \dfrac{\partial T(\vec{x},t)}{\partial t} = \lambda \Delta T(\vec{x},t) + \psi \qquad \forall \vec{x} \in \mathcal{D} & (1) \\ \lambda \vec{\nabla} T(\vec{y},t).\vec{n}(\vec{y}) = h(T_{\text{F}} - T(\vec{y},t)) - h_r \left(T(\vec{y},t) - \displaystyle\int_{\mathcal{H}(\vec{y})} \frac{|\vec{\omega}.\vec{n}(\vec{y})|}{\pi} T_{\text{R}}(\vec{\omega},\vec{y},t) d\vec{\omega} \right) \qquad \forall \vec{y} \in \partial\mathcal{D}_{\text{F}} & (2) \\ T(\vec{y},t) = T_{\text{B}} \quad \forall \vec{y} \in \partial\mathcal{D}_{\text{top}}; \quad \lambda \vec{\nabla} T(\vec{y},t).\vec{n} = \varphi \quad \forall \vec{y} \in \partial\mathcal{D}_{\text{bot}}; \quad \lambda \vec{\nabla} T(\vec{y},t).\vec{n} = 0 \quad \forall \vec{y} \in \partial\mathcal{D}_{\text{lat}} & (3) \\ T(\vec{x},0) = T_{\text{I}} \qquad \forall \vec{x}, \vec{y}; t = 0 & (4) \end{cases}$$

The solid properties are homogeneous and independent of temperature: λ is the thermal conductivity, ρ, the density, c, the specific heat capacity and h, the convective heat transfer coefficient. A power density ψ exists uniformly throughout the whole solid phase (cf. Eq. (1)). T_F is the temperature of the fluid surrounding the solid domain and T_{BR} is the ambient radiance temperature. The solid is assumed opaque with an uniform emissivity ϵ. Radiative heat transfer can be linearized around a reference temperature T_{ref}, hence $T^4 \approx T_{\text{ref}}^4 + h_{\text{R}}(T - T_{\text{ref}})$ with $h_{\text{R}} = 4\epsilon\sigma T_{\text{ref}}^3$. σ is the Stefan-Boltzmann constant, $\vec{n}(\vec{y})$ is the normal heading towards the solid, $\vec{y}$ is a location at the solid/fluid boundary and $T_{\text{R}}(\vec{\omega},\vec{y},t)$ is modeled as a mean radiative temperature seen at the solid/fluid interface due to radiative exchanges through the fluid phase (cf. Eq. (2)). A solid temperature T_B is imposed at the top surface $\mathcal{D}_{\text{top}}$, a flux density φ is imposed at the bottom surface $\mathcal{D}_{\text{bot}}$, lateral faces of the channel corresponding to $\partial\mathcal{D}_{\text{lat}}$ interface are insulated (cf. Eqs. (3)) and the initial temperature T_I is uniform (cf. Eq. (4)).

To evaluate a local temperature $T(\vec{x}_{\text{obs}}, t_{\text{obs}})$ at a given location $\vec{x}_{\text{obs}}$ and a given time t_{obs} with a MC algorithm, N thermal paths γ_i are sampled and each of these paths provides a MC weight w_{γ_i}. These weights are used to produce an estimate m of the sought quantity $T(\vec{x}_{\text{obs}}, t_{\text{obs}})$ and, at the same time, a statistical error s associated with the estimate (cf. Eqs. (5)).

$$\left\{\begin{array}{l} T(\vec{x}_{\text{obs}}, t_{\text{obs}}) \approx m = \frac{1}{N}\sum_{i=1}^{N} w_{\gamma_i} \\ s = \frac{1}{\sqrt{N}}\left(\frac{1}{N}\sum_{i=1}^{N} w_{\gamma_i}^2 - \left(\frac{1}{N}\sum_{i=1}^{N} w_{\gamma_i}\right)^2\right)^{\frac{1}{2}} \end{array}\right. \tag{5}$$

During a single MC algorithm a thermal path can alternatively undergo a conductive path, a radiative path or end at the solid/fluid boundary with a convective path [Fournier, 2016]. Two examples of thermal conductive-convective-radiative paths are represented in Figure 2. The first path successively follows conductive paths in the solid phase, radiative paths in the fluid phase and finally ends at the solid/fluid boundary with the convective boundary condition. Hence, the stored weight w_{γ_i} is the temperature T_{F} with a contribution of the power density Ψ proportional to all paths performed in the solid phase. The second path also follows the conductive and radiative paths to finally end encountering the initial condition temperature T_{I}. The corresponding stored weight w_{γ_i} is thus T_{I} with a contribution of the power density Ψ and a contribution of the flux density φ. The random conductive-convective-radiative paths generated during a MC algorithm can be used to estimate the contributions of each heat source involved in the thermal problem. With regard to Green's theory, six sources $\mathcal{S}$ are involved in this configuration (cf. Figure 2): $\mathcal{S} = \{T_{\text{I}}, T_{\text{B}}, T_{\text{F}}, T_{\text{BR}}, \psi, \varphi\}$. The MC weight w_{γ_i} can be expressed as follows :

$$w_{\gamma_i} = T_{\text{end}} + \beta_{\psi}(\vec{x}_{\gamma_i,\psi})\ \psi + \beta_{\varphi}(\vec{x}_{\gamma_i,\varphi})\ \varphi \tag{6}$$

Four different known temperatures may be encountered at the end of the path: $T_{\text{I}}, T_{\text{B}}, T_{\text{F}}$ and T_{BR} matching respectively indexes end $= 0, 1, 2$ and 3. Along the path two contributions are stored, respectively $\beta_{\psi}(\vec{x}_{\gamma_i,\psi})$ for the power density ψ and $\beta_{\varphi}(\vec{x}_{\gamma_i,\varphi})$ for the flux density φ. Using the previous equation in Eq. (5), the temperature $T(\vec{x}_{\text{obs}}, t_{\text{obs}})$ can be expressed as follows:

$$T(\vec{x}_{\text{obs}}, t_{\text{obs}}) \approx \sum_{\text{end}=0}^{3} \underbrace{\frac{N_{\text{end}}}{N}}_{m_{\text{end}}} T_{\text{end}} + \underbrace{\frac{1}{N}\sum_{i=1}^{N} \beta_{\psi}(\vec{x}_{\gamma_i,\psi})}_{m_{\psi}}\ \psi + \underbrace{\frac{1}{N}\sum_{i=1}^{N} \beta_{\varphi}(\vec{x}_{\gamma_i,\varphi})}_{m_{\varphi}}\ \varphi \tag{7}$$

The SMC algorithm uses a single MC calculation to provide the estimates $m_{\mathcal{S}} = \{m_{\text{I}}, m_{\text{B}}, m_{\text{F}}, m_{\text{BR}}, m_{\psi}, m_{\varphi}\}$ of each source in order to output the temperature $T(\vec{x}_{\text{obs}}, t_{\text{obs}})$ and the associated statistical error s as linear functions (propagators) of the sources $\mathcal{S}$.

RESULTS

As represented in Figure 3, nine different geometries were studied with three different values of porosity (40%, 60% and 80%) and, for each of these values, three different mesh refinements i.e number of triangles forming the mesh (10^4, 10^5 and 10^6). For each case, a single SMC calculation was performed to estimate the transfer function at the near center of the geometry ($\vec{x}_{\text{obs}} = [0.73L_x, 0.55L_y, 0.34L_z]$) at steady state. The function was then used to estimate the probe temperature $T(\vec{x}_{\text{obs}}, t_{\text{obs}})$ for any value of the six sources $\mathcal{S}$ involved. For each value of porosity, the computation time needed to estimate the function (t_{SMC}) and the computation time to estimate a temperature with the obtained function (t_{fun}) according to the number of triangles forming the mesh are represented in Figure 4. Because of the generation of various conductive-convective-radiative paths in these complex geometries, the computation time t_{SMC} needed to compute the transfer function with a single SMC calculation is approximately 10^3s (almost 17 min). Thereafter, the computation time t_{fun} needed to compute the temperature with the transfer function such as expressed in Eq. (7) only requires a few milliseconds (0.1-0.4ms), which is logical, due to the succession of elementary operations used to compute the solution. The computation with the transfer function is approximately 10^4 faster than the time needed to estimate the function with a SMC calculation (t_{SMC} also corresponds to the time needed to estimate a probe temperature with a standard MC computation). Hence, computational times to estimate the transfer function and perform a probe temperature estimation with the obtained function are dependent neither on the porosity value nor on the mesh refinement.

CONCLUSION

The SMC method has shown insensitivity of computation time to mesh refinement and to geometrical complexity on an example of a typical 3D porous medium corresponding to a heat exchanger. Computation times needed to estimate the transfer function and compute a probe temperature for any heat source values with the function are dependent neither on the value of the medium porosity nor on the number of triangles forming the surfacic mesh. The deployment of the SMC algorithm was possible thanks to recent advances in Green's formulation with the generation of thermal conductive-convective-radiative paths and the implementation of combined heat transfer calculations with MC

methods in complex geometries, through the development of computer graphics libraries (*Stardis* : https://www.meso-star.com/projects/stardis/stardis.html). Given these insensitivity benefits, the SMC transfer function is useful in the perspective of optimization problems where the local temperature needs to be evaluated a large number of times (control algorithms, inversion problems). In the work outlined here, the transfer function depends linearly on the heat sources of the problem. As regards theoretical developments to extend the MC calculation and therefore, in the longer term the SMC method, work has been undertaken to take into account the heat advection term (added in Eq. (1) with a known velocity field) [Ibarrart, 2018] and on the implications of not linearizing radiative transfer when coupling to the heat transfer equation [Tregan, 2020].

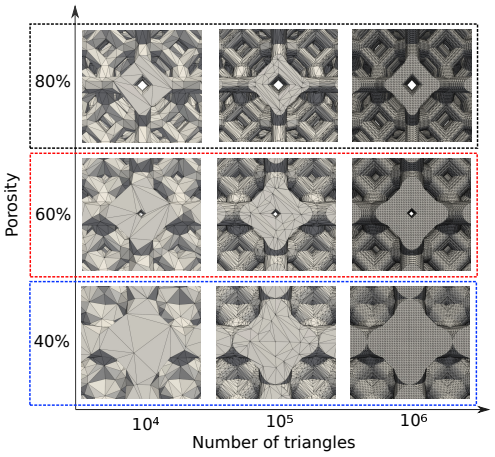

Figure 3: Different values of porosities and number of triangles for the porous medium.

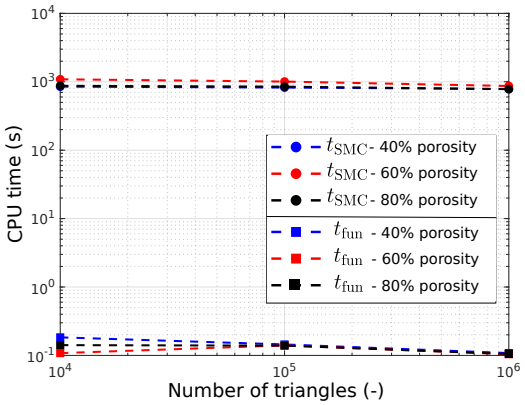

Figure 4: Computation times t_{SMC} and t_{fun} according to the number of triangles for different values of porosities (Intel Core i5 - CPU @ 1.60GHz x 8, RAM : 16Gb).

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