NUMERICAL SIMULATION OF FAILURE PREDICTION FOR CERAMIC TOOLS: COMPARISON WITH FORGING TEST RESULTS

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Abstract: A fracture prediction criterion for brittle materials has been introduced in the POLLUX finite element code, in order to predict the risk-of-rupture of ceramic tools during a forging operation. POLLUX is a software dedicated to the simulation of forging operations, initially developed by INSA (Lyon). The chosen probabilistic fracture model is based on the weakest link theory and on the statistical theory of Weibull. A surface approach or a volume approach can be retained, from the type of critical flaws in the ceramic. Two different criteria are available in order to characterise the stress state considering the tension normal stresses and neglecting the compression stresses.
An identification procedure of critical flaw type is presented for a particular ceramic material. Statistical parameters of ceramic fracture have been experimentally determined using bending tests performed in environmental conditions close to the forging ones. A constitutive equation of the workpiece material has been proposed, issued from torsion tests.
In order to validate the model in the case of ceramic tools subjected to multiaxial stress states, a particular configuration has been defined to compare the simulation predictions with the experimental results. A forging test has been then developed, in which a billet of superalloy is formed in a ceramic tool up to its fracture at the temperature of 1423K.
The experimental distribution of tool fracture, according to the strain of the billet, are in good agreement with fracture predictions computed by the simulation.

Keywords: finite element method, fracture prediction, ceramics, forging.

1. INTRODUCTION

Isothermal forging is a very effective forming process for some refractory alloys like nickel-based materials. This technique should allow the production of near net shape parts, directly from simple shape preforms. However, the process is based on the superplastic properties of the forming materials, and it must be carried out at high temperature and at low strain rate. This means that the material for the forging dies, which will be used at the same temperature as the workpiece, must conserve high mechanical properties at high temperature. The forging tools are actually made in Mo-based alloys, but the high reactivity of this element with oxygen imposes to carry out the process under an inert gas. The use of ceramic dies should be a less expensive way, as the good oxidation resistance of these materials does not require a controlled atmosphere.

However, ceramics are inherently brittle. This characteristic leads in particular to a wide variation in the material strength. Usual design techniques are therefore not suited to ceramic components, and statistic and reliability analysis are needed. Statistical fracture models, based on the weakest link theory, have been previously developed. The first probabilistic approach used to account for the scatter in fracture strength of brittle materials was introduced by Weibull [1]. This model has been initially proposed for constant uniaxial stress fields. New criteria were later added to extend the Weibull approach to multiaxial stress fields [2,3]. An other model, developed by Batdorf, combines the weakest link theory and the linear elastic fracture [4].

Except in some simple cases, it is necessary to use numerical simulation in order to predict the risk of failure of ceramic parts with these models. Some studies have been previously conducted, in which brittle fracture models were coupled to finite element simulations in order to calculate the failure probability of particular ceramic structures. In some cases, a post processor has been developed for a specific software, to analyse the probabilistic behaviour of simple ceramic structures [5] or of uniaxially loaded ceramic composites [6]. However, most of the studies in this field have been made by the National Aeronautics and Space Administration [7-9]. A computerised design program for reliability evaluation of ceramic components has been developed from this research work. This software can be coupled to the ANSYS, ABAQUS, NASTRAN and COSMOS commercial finite element programs. Nevertheless, any program previously quoted is not convenient for the simulation of the forging process, as large strains take place in the workpiece.

The main objective of this study is to develop a finite element model for the failure prediction of ceramic forging dies as a function of the service conditions in order to improve the design of these components.

2. DESCRIPTION OF THE FAILURE PREDICTION SOFTWARE

2.1. Brittle fracture modelling

The wide variation of the strength measured for brittle materials is due to the nature and distribution of intrinsic microscopic flaws. These flaws are unavoidably present as a result of the material manufacturing and machining operations, or of a damage process. The distinction must be made between volume and surface flaws, which lead to different strength distributions. In the case of a coarse material microstructure, the volume flaws are generally more critical than the surface ones. However, the surface flaws may be more critical for example in the case of rough machining or of oxidised surface states. The identification of the nature of critical flaws allows one to choose the analysis approach. If a surface one is chosen, the volume flaws are assumed not to be responsible for the fracture.

As the flaw distribution cannot be determined, some statistical functions are used to describe the strength variability of a component. The Weibull model has been chosen in the present study because the statistical parameters are experimentally easy to determine. The Weibull theory associates a strength σ_c to each elementary volume, Ve (or elementary surface Se for the surface approach). The strength of the material is characterised by the distribution function of σ_c, denoted $F(\sigma)$, which is defined as the number of all elementary volumes (surfaces) having $\sigma_c \leq \sigma$, divided by the total volume (external surface). This function also gives the probability P of randomly choosing an elementary volume (surface) having a strength equal to or less than σ, or also the probability that this volume fails under the stress σ:

$$P(\sigma_c \leq \sigma) = F(\sigma) \quad (1)$$

Any distribution function may be written in the form:

$$F(\sigma) = 1 - \exp(-\phi(\sigma)) \quad (2)$$

where $\phi(\sigma)$ must be a positive and non decreasing function, vanishing at a value σ_u, called the threshold stress.

The advantage of this formula appears when considering the failure probability of the structure constituted by all the elementary volumes (surfaces), using the weakest link theory. When supposing that the total volume V (surface S) fails if any one of its parts fails, then the probability of non failure P_{NF}, under an uniform uniaxial load σ, is equal to:

$$P_{NF} = (1-P)^n = \exp(-n\phi(\sigma)) \quad (3)$$

where n is the number of elementary volumes (surfaces) of the structure (V=nVe or S=nSe)

The failure probability P_n of the structure takes then the simple form:

$$P_n = 1 - \exp(-n\phi(\sigma)) \quad (4)$$

Weibull has chosen a power form for the function $\phi(\sigma)$, which is the most simple function satisfying the general conditions previously described:

$$\phi(\sigma) = \left(\frac{\sigma - \sigma_u}{\sigma_0} \right)^m \quad (5)$$

where σ_u, σ_0 and m are parameters depending on the material characteristics.

The threshold stress σ_u is usually taken equal to zero. The two other parameters can then be easily identified by comparison with results of fracture tests.

In the case of a non uniform uniaxial load σ, the failure probability takes the integral form:

$$P_v = 1 - \exp\left(-\frac{1}{Ve}\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dV \right)$$
(6.1)

for the volume approach, or

$$P_s = 1 - \exp\left(-\frac{1}{Se}\int_S \left(\frac{\sigma}{\sigma_0}\right)^m dS \right)$$
(6.2)

for the surface approach.

The previous relationship can be generalised to multiaxial stress states. Two strategies are available: the first consists in considering an equivalent stress which characterises the loading at each point. This criteria leads sometimes to unsafe estimates of the failure probability; the second consists in assuming the independence of the actions of each principal tensile stress [2].

Taking into account the first approach, Weibull proposed to calculate an equivalent stress by averaging the normal tensile stresses in all directions. This model leads to a complex integral formulation, which may increase the computation time.
In order to obtain shorter computation times, only the maximal normal stress has been considered in our model, as other investigators did [5]. This variable is also the maximal principal stress at each point of the structure.

In the second approach, the fracture probability depends on all the tensile principal stresses. The independence of actions of the principal stresses means that the reliability of the structure is the product of the reliability of the individual principal stresses acting independently. The fracture probability is then given by the following relationship:

$$P_v = 1 - \exp\left(-\frac{1}{Ve}\int_V \left(\left(\frac{\sigma_1}{\sigma_0}\right)^m + \left(\frac{\sigma_2}{\sigma_0}\right)^m + \left(\frac{\sigma_3}{\sigma_0}\right)^m \right) dV \right)$$

for the volume approach, or
(7.1)

$$P_s = 1 - \exp\left(-\frac{1}{Se}\int_S \left(\left(\frac{\sigma_1}{\sigma_0}\right)^m + \left(\frac{\sigma_2}{\sigma_0}\right)^m + \left(\frac{\sigma_3}{\sigma_0}\right)^m \right) dS \right)$$

for the surface approach,
(7.2)

where σ_1, σ_2 and σ_3 are the positive principal stresses or take the zero value for the compressive principal stresses.

It is interesting to note that in the both models, the compressive stresses are not taken into account. The fracture is supposed to be always the result of tensile stresses. This choice has been made because the compressive fracture strength of ceramics is 3 or 4 times higher than the tensile strength. It is so very unlikely that structure fails under the effect of compressive stresses.

2.2. Numerical introduction of the model
The model previously described has been introduced as a subroutine in the POLLUX finite element code [10]. POLLUX is a specialised software developed by INSA-Lyon [11] in order to predict the stress state in the workpiece and the surrounding tools during a forging process. First, the thermomechanical analysis is achieved, leading to the calculation of the stress tensor at each point of the tools, which are supposed to show a perfectly elastic behaviour. Then, the failure probabilities of the tools are determined, using the Weibull parameters and the criteria chosen by the user.

If Ve or Se are arbitrary taken equal to 1, the discrete decomposition of the models gives:

$$P_v = 1 - \exp\left(-\sum_{iel}\sum_{ig} \left(\frac{\sigma_1}{\sigma_0}\right)^m V(ig) \right)$$
(8.1)
for the volume approach, or

$$P_s = 1 - \exp\left(-\sum_{inc} \left(\frac{\sigma_1}{\sigma_0}\right)^m S(inc) \right)$$
(8.2)
for the surface approach,

if the maximal principal stress criteria (MPS) is chosen, and:

$$P_v = 1 - \exp\left(- \sum_{iel} \sum_{ig} \sum_{i=1}^{3} \left(\frac{\sigma_i}{\sigma_0} \right)^m V(ig) \right) \quad (9.1)$$

for the volume approach, or

$$P_s = 1 - \exp\left(- \sum_{inc} \sum_{i=1}^{3} \left(\frac{\sigma_i}{\sigma_0} \right)^m S(inc) \right) \quad (9.2)$$

for the surface approach,

if the principle of the independent actions (PIA) is chosen.

σ_1, σ_2, σ_3 are the principal stresses in the Gauss point ig of element iel or in the contour node inc($\sigma_1 \geq \sigma_2 \geq \sigma_3$) or are equal to zero for the compressive principal stress.
V(ig) is the volume associated to the Gauss point ig,
and S(inc) is the surface associated to the contour node inc.

The elementary failure probability DPr, or local risk-of -rupture, is also calculated at each Gauss point in the case of the volume approach, and interpolated to the nodes. This variable corresponds to the fracture probability of an elementary volume of constant size equal to 1, subjected to the stress state of the considered point. The values can be stored in a data file for graphical display of the critical regions.
An other data file, containing the failure probability of the overall structure as a function of the time or tool displacement, is also generated.
At the end of each increment, the program compares failure probability of the tools to the maximal permissible probability, chosen by the user. The computation is interrupted when this limit or the maximal tool course is achieved.

3. RELIABILITY ANALYSIS PROCEDURE FOR CERAMIC FORGING TOOLS

3.1. Identification of critical flaw type

Reliability analysis of ceramic structures with the software previously described, first requires to choose a failure criteria corresponding to the material and the use conditions. If the choice between MPS and PIA criteria can not be made by considering physical observations, the choice of the approach type (surface or volume) must be based on these observations.

In the case of the forging process with ceramic tools, the material strength is modified, because of damage inherent to the use conditions. Two types of flaws can be created: surface flaws, essentially produced by the oxidation of the material at the forging temperature, and internal flaws (microcracks), which can be formed because of the mechanical loading of the tool or because of the thermal cycles.
In order to identify the critical flaw type formed during the forging process, we have compared the fracture strength of the initial material and of the material subjected to a damaging factor (oxidation, mechanical or thermal cycles). The failure probability of the material, as a function of the stress, has been determined using four point bending tests (see section 3.2), at room temperature. Fig.1 shows the experimental points and the corresponding interpolated Weibull functions for the different states of the material.
First, it is to be noticed that the oxidised material has a weaker strength than initial material. Then it appears that mechanical and thermal cycles have no significant effects on the material characteristics. The strength distribution of the material after mechanical cycles is indeed very close to the initial material one, and the strength distribution of the material after oxidation and thermal cycles is approximately the same as for the uncycled oxidised material.
This means that the surface flaws, created by the oxidation of the material, are more critical than the initial flaws, or the internal flaws resulting of the cyclic loading. According to this result, the surface approach must be chosen for the calculation of the ceramic tools reliability during the forging process.

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Fig.1. Fracture strength distribution of the initial and of the damaged material.

3.2. Determination of statistic parameters

The second step of the reliability analysis of the ceramic tool consists in the experimental determination of the statistical parameters of the Weibull model corresponding to the material and to the processing conditions.

Four points bending tests (Fig.2) have then been carried out at 1423K on a set of N specimens previously oxidised at 1423K during 50 hours. The failure probability of each sample is given by the formula:

$$P_{ex} = \frac{i}{N+1} \qquad (10)$$

where i is the rank of the maximal stress σ_{max} in the corresponding specimen at the fracture point. σ_{max} is given by:

$$\sigma_{max} = \frac{3}{2}\frac{F(L-l)}{bh^2} \qquad (11)$$

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Fig.2. Four point bending tests

The Weibull distribution corresponding to the bending tests, in the case of surface critical flaws, can be expressed as follow:

$$P_{th} = 1 - \exp\left(-S_{eff}\left(\frac{\sigma_{max}}{\sigma_u}\right)^m\right) \qquad (12)$$

where S_{eff} is the effective surface, determined from the surface in tension, S, of a specimen:

$$S_{eff} = \int_S \left(\frac{\sigma}{\sigma_{max}}\right)^m dS \qquad (13)$$

Eq. (12) can also be written:

$$\ln\left(\ln\left(\frac{1}{1-P_{th}}\right)\right) = \ln\left(\frac{S_{eff}}{\sigma_u^{\ m}}\right) + m.\ln(\sigma_{max}) \qquad (14)$$

The Weibull plot, which expresses the evolution of ln(ln(1/1-P)) as a function of $\ln(\sigma_{max})$, is usually used to represent the experimental results.

The Weibull parameters are estimated by fitting the equation (14) in the experimental points of a Weibull plot, using the least squares method (Fig.3).

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Fig.3. Weibull's plot of the four point bending strength at 1423K of ceramic specimens previously oxidised at 1423K during 50 hours.

The Weibull parameters m and σ_u obtained are respectively equal to 12.4 and 580 MPa.

3.3. Determination of the workpiece rheology

The further set of data required for the reliability analysis is the rheological behaviour of the workpiece material. In the forging conditions, the superalloys have a superplastic behaviour. In consequence, the flow stress depends on the strain rate.

In order to determine the constitutive equation of this material, torsion tests have been carried out at the forging temperature, with 3 different strain rates ranging from 0.03 to 0.3 s^{-1}. During the test, the experimental torque Γ is measured as a function of the turn number T. The equivalent strain rate $\dot{\bar{\varepsilon}}$ and strain $\bar{\varepsilon}$, calculated at the periphery of the specimen, are given by:

$$\dot{\bar{\varepsilon}} = 2\pi\frac{\dot{T}R}{\sqrt{3}b} \qquad \text{with } \dot{T} = \frac{dT}{dt} \qquad (15)$$

$$\bar{\varepsilon} = 2\pi\frac{TR}{\sqrt{3}b} \qquad (16)$$

where R and b are respectively the radius and the length of the specimen.

The Von Mises equivalent stress $\bar{\sigma}$ is given by:

$$\bar{\sigma} = \frac{\Gamma\sqrt{3}}{2\pi R}\left(3 + \tilde{n} + \tilde{m}\right) \qquad (17)$$

where $\tilde{n}$ and $\tilde{m}$ represent the sensitivity of the torque to the strain and to the strain rate.

The evolution of $\bar{\sigma}$ versus $\bar{\varepsilon}$ are plotted in Fig.4 for the different strain rates.

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Fig.4. Stress-strain curves obtained from torsion test at 1423K and calculated from the model.

The curves show a stress peak ($\bar{\sigma}_p$), followed by a softening and a stabilisation of the stress values ($\bar{\sigma}_s$).
According to Berdin et al. [12], the stress peak observed in this type of material at the low strain levels is due to a dynamic recrystallisation phenomenon. The material behaviour can be approximated by the model:

$$\bar{\sigma} = (1-\alpha)\bar{\sigma}_p + \alpha\bar{\sigma}_s \quad (18)$$

with $\bar{\sigma}_p = A_p\dot{\bar{\varepsilon}}^{mp}$, $\bar{\sigma}_s = A_s\dot{\bar{\varepsilon}}^{ms}$ (19)

where the different constants have been determined by fitting the model to the experimental results.
mp and ms are the strain rate sensitivity coefficients, respectively equal to 0.22 and 0.15.
A_p and A_s depend only on the temperature and are respectively equal to 150.1 and 109.6.
α represents the recrystallized fraction, given by an Avrami type equation [12]. Figure 4 shows a good agreement between the experimental results and the curves calculated from this model.

4. CONFRONTATION WITH FORGING-FRACTURE TEST RESULTS

4.1. Description of the forging test

In order to validate the model and the approach previously described, fracture tests have been carried out on ceramic tools in near forging conditions. The results were compared to the numerical results of the tool reliability.
Fig.5 shows the samples chosen for this confrontation. The ceramic and the workpiece were heated until 1423K in an induction furnace. During the test under isothermal conditions, a superalloy workpiece is pressed by a punch in the ceramic specimen until its fracture. The forging load and the punch displacement are measured all along the tests.

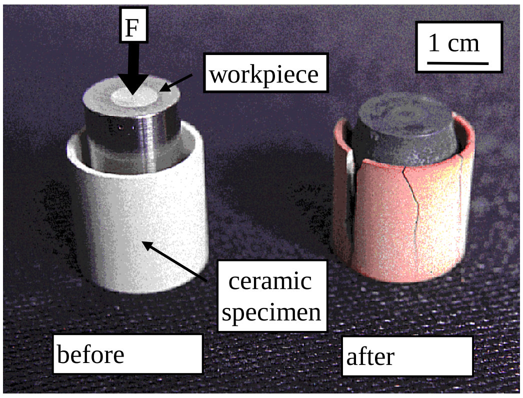

Fig.5. Ceramic specimen and workpiece used for the fracture tests

After the tests, realised in the same conditions on a set of ceramic specimens, the failure probability calculated from Eq. (10) is expressed as a function of the punch displacement. A program of 18 fracture tests has been planned to obtain an experimental distribution of ceramic fracture.

4.2. Failure probability calculation

At the moment, only the two first tests have been carried out, but encouraging results have been obtained. A good agreement between the experimental results and the numerical simulation predictions is indeed observed.
First, the calculation of the stress state in the specimen has shown that the orthoradial stress is the maximal component of the stress tensor in the ceramic (Fig.6). This predicts that at the fracture, the cracks in the ceramic specimen must be longitudinal. This has been verified experimentally and can clearly be observed on Figure 5.

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Fig.6. Orthoradial stress distribution obtained from the forging test simulation after a punch displacement of 4 mm.

Secondly, the failure distribution of the ceramic specimens, expressed as a function of the punch displacement, obtained from the numerical calculation are presented on Fig.7. The experimental values measured during forging tests are very closed to the calculated ones.

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Fig.7. Experimental and calculated failure probability of the ceramic specimen.

Both MPS and PIA criteria available for the calculation have been tested, in order to determine the more efficient for the present case. Nevertheless, the choice seems to be difficult, because of the weak difference obtained from the both criteria. This is due to the fact that for the ceramic geometry chosen, one of the principal stresses is always predominant, and the two others have no significant effects on the risk-of-rupture. The more appropriated criteria will probably be easier to determine when considering the totality of the 18 experimental tests.

5. CONCLUSION

A fracture prediction program for brittle materials has been introduced in the finite element code POLLUX, in order to predict the risk-of-rupture of ceramic tools during high temperature forging operations.

For an efficient use, the following characteristics must be known:

- the type of the critical flaws in the ceramic;
- the statistical parameters of the fracture strength distribution of the ceramic;
- the metal workpiece rheology.

These characteristics can be measured by classical mechanical tests (bending, torsion or tension tests).

The software is then a very useful prediction tool for the fracture of ceramic parts:

- it permits to predict the failure probability of a ceramic tool with a given geometry, as a function of its use conditions.
- it can also give elements to predict the localisation and the orientation of the fracture cracks. Such a software can so be very helpful for the design of the geometry and of the dimension of large ceramic tools.

The comparison between the numerical simulation results and the forging fracture tests has shown a good agreement. Complementary experimental tests will be performed in a mean future in order to have a more complete comparison , particularly in terms of the statistical distribution of the fracture loads and displacements. These results are expected to be very helpful to choose the more appropriated fracture criteria (PIA or MPS) for the ceramic material used in the present study. Other failure criteria could also be considered, for example taking into account the shear stresses in the ceramic. Future developments of the software forecast the introduction of this type of criteria.

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